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$$= -\frac{t}{n}(\sin\frac{1}{2}\pi n + \sin\frac{3}{2}\pi n) = 0.$$

$$\therefore f(x) = \frac{5}{2}t - (t/\pi)(4\sin x + 2\sin 2x + \frac{4}{3}\sin 3x + \frac{4}{5}\sin 5x + \frac{4}{7}\sin 7x + \frac{4}{9}\sin 9x + \dots)$$

$$\therefore V = e^{-hT}[\frac{5}{2}t - (t/\pi)(4\sin xe^{-kT} + 2\sin 2xe^{-2kT} + \frac{4}{3}\sin 3xe^{-3kT} + \frac{4}{5}\sin 5xe^{-5kT} + \dots)]$$

AVERAGE AND PROBABILITY.

108. Proposed by A. H. HOLMES, Brunswick, Me.

Required the average area of the quadrilateral whose sides are a , b , c , and d .

I. Solution by G. B. M. ZERR, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $ABCD$ be the quadrilateral, $AB=a$, $AD=b$, $BC=c$, $CD=d$, $a > b > c > d$, $BE=v$, $DE=u$.

When the quadrilateral is convex, the average area is, (the average area of ABD) + (the average area of BCD).

When the quadrilateral is concave, the average area is, (the average area of ABD) - (average area of $BC'D$).

Since BCD is equal to $BC'D$, the average area required is the average area of $ABD = \Delta$. Area $ABD = \frac{1}{2}ab\sin A$.

$$\text{Now } DB = \sqrt{u^2 + v^2}.$$

$$\therefore \cos A = (a^2 + b^2 - u^2 - v^2) / 2ab.$$

$$\therefore \text{Area } ABD = \frac{1}{4}\sqrt{[4a^2b^2 - (a^2 + b^2 - u^2 - v^2)^2]}.$$

$$\text{But } v^2 = a^2 - (b-u)^2.$$

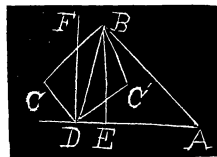
$$\therefore \text{Area } ABD = \frac{1}{2}b\sqrt{[a^2 - (b-u)^2]} = C.$$

The limits of u are 0 and $[(c+d)^2 + b^2 - a^2] / 2b = u'$.

$$\therefore \Delta = \frac{\int_0^{u'} C du}{\int_0^{u'} du} = \frac{b}{2u'} \int_0^{u'} \sqrt{[a^2 - (b-u)^2]} du.$$

$$\therefore \Delta = \frac{b^2}{(c+d)^2 + b^2 - a^2} \left[\frac{b}{2} \sqrt{a^2 - b^2} + \frac{a^2}{2} \sin^{-1} \frac{b}{a} - \frac{a^2 + b^2 - (c+d)^2}{8b^2} \right]$$

$$\sqrt{[4a^2b^2 - [a^2 + b^2 - (c+d)^2]^2]} - \frac{a^2}{2} \sin^{-1} \left(\frac{a^2 + b^2 - (c+d)^2}{2ab} \right) \Bigg].$$



II. Solution by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy, Defiance College, Defiance, O.

Let $ABCD$ be the quadrilateral, side $AB=a$, $BC=b$, $CD=c$, $DA=d$, $\angle ABC=\theta$, and $\angle CDA=\phi$.

Suppose the vertices of the quadrilateral to be movable, or hinged; then the side BC may make a complete revolution about B as a center so long as $(c+d)$ is not less than $(a+b)$. That is, for $(c+d)$ not less than $(a+b)$, the angle θ will vary uniformly from 0° to 360° ; but as soon as the side BC has moved below the side AB produced, we have hour-glass quadrilaterals composed of two practically isolated triangles which are not to be considered in finding the average area of the quadrilateral (a, b, c, d) . We are, therefore, constrained to regard θ as varying uniformly from 0° to 180° ; but if $(c+d) < (a+b)$, which becomes certain when the sides of the quadrilateral (a, b, c, d) are numerically expressed, we are constrained to regard θ as varying uniformly from 0° to $\cos^{-1}\{[a^2+b^2-(c+d)^2]/2ab\}$.

From the diagram it is evident that $Q=(\triangle ABC + \triangle CDA) = \frac{1}{2}(ab\sin\theta + cd\sin\phi)$. Now $AC = \sqrt{a^2+b^2-2ab\cos\theta}$, and

$$\cos\phi = \frac{c^2+d^2-(a^2+b^2-2ab\cos\theta)}{2cd}.$$

[For the sake of brevity, put $m=c^2+d^2-a^2-b^2$, $n=4c^2d^2-m^2$, $p=n/4a^2b^2$, and $q=m/ab$].

$$\begin{aligned} \therefore \sin\phi &= \sqrt{1 - \left(\frac{m+2ab\cos\theta}{2cd}\right)^2} = \sqrt{\frac{(4c^2d^2-m^2)-4abm\cos\theta-4a^2b^2\cos^2\theta}{4c^2d^2}} \\ &= \frac{1}{2cd} \sqrt{(n-4abm\cos\theta-4a^2b^2\cos^2\theta)} = (ab/cd) \sqrt{(p-q\cos\theta-\cos^2\theta)} \\ &= (ab/cd) \sqrt{[(p+\frac{1}{4}q^2)-(\frac{1}{2}q+\cos\theta)^2]}. \end{aligned}$$

$$\text{Also, } Q = \frac{1}{2}ab\{\sin\theta + \sqrt{[(p+\frac{1}{4}q^2)-(\frac{1}{2}q+\cos\theta)^2]}\}.$$

Representing $\cos^{-1}\{[a^2+b^2-(c+d)^2]/2ab\}$ by θ_1 , the expression for the average area of the quadrilateral on the hypothesis that the interior angle at B vary uniformly from 0 to θ_1 becomes

$$Q_B = \frac{ab}{2\theta_1} \int_0^{\theta_1} \{\sin\theta + \sqrt{[(p+\frac{1}{4}q^2)-(\frac{1}{2}q+\cos\theta)^2]}\} d\theta.$$

Similar operations give Q_C , Q_D , and Q_A ; and, therefore, the required average area of the quadrilateral (a, b, c, d) becomes $Q = \frac{1}{4}(Q_A + Q_B + Q_C + Q_D)$. Slight modifications, in signs, etc., may be occasioned by quadrilaterals having a re-entrant angle.

